APPENDIX

A. Properties of Bézier curves

A Bézier curve $b : [0,1] \to \mathbb{R}$ of degree n_b is defined as a linear combination of Bernstein polynomials, [26, p. 144],

$$b(t) := \sum_{k=0}^{n_b} \beta_k B_{k,n_b}(t), \tag{28}$$

where B_{k,n_b}, $k = 0, 1, \ldots, n_b$ are the Bernstein polynomials of degree n_b, and the coefficients $\beta_0, \beta_1, \ldots, \beta_{n_b}$ are referred to as control points.

1) Convex hull property: The convex hull property of Bézier curves provides constraint fulfillment guarantees by checking the constraint only at its control points. The fact that the Bernstein polynomials form a partition of unity implies

$$(t, b(t)) \in \text{conv}(\{(k/n_b, \beta_k), k = 0, \ldots, n_b\}), \tag{29}$$

for all $t \in [0,1]$, [26, p. 146].

2) Refinement: Using de Casteljau's algorithm, [26, p. 151], the Bézier curve b, defined on the interval $[0,1]$, can be efficiently split into two Bézier curves of the same degree, defined over the intervals $[0,\alpha]$ and $[\alpha,1]$ with $0 < \alpha < 1$.

3) Multiplication: Consider the Bézier curves b and c of degree n_b respectively n_c, with the coefficients β_k respectively γ_k. The coefficients α_k of the Bézier curve $a(t) = b(t)c(t)$, $t \in [0,1]$ of degree $n_b + n_c$ are then given by

$$\alpha_i = \sum_{j=\max(0,i-n_b)}^{\min(n_c,i)} \frac{\binom{n_c}{j}\binom{n_b}{i-j}}{\binom{n_b+n_c}{i}} \gamma_j \beta_{i-j}. \tag{30}$$

4) Approximation of a convex function: In addition, the following result will be used.

Proposition 6.1: (from [32]) Let $f : [0,1] \to \mathbb{R}$ be an arbitrary continuous convex function. For any integer $n_b > 0$ it holds that

$$\sum_{k=0}^{n_b} f(k/n_b) B_{k,n_b}(t) \geq f(t) \tag{31}$$

for all $t \in [0,1]$. Equality holds in the limit as $n_b \to \infty$ (uniformly in $t \in [0,1]$).

Proof: See [32, p. 255, p. 259]. ∎